Avanti Learning Centre

2014 - 2016

# P8. Mechanical Properties of Solids

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## Elasticity I - Introduction

#### Pre-reading Exercise (10 mins + 5 mins GD)

**Solution:** By definition it is a perfectly elastic body.

**Solution:** Stress ( where F = tensile force and A = Area of cross section)

**Solution:** In the region OA, stress strain i.e. Hooke’s law holds good

**Solution:** Point B is called as the Yield point. It is also known as the elastic limit.

**Solution:** Point D is called as the breaking point. It represents the ultimate tensile strength of the material.

**Solution:**

Here

Young’s Modulus (Y)

Force

From definition of Young’s Modulus

#### 

#### In-class Exercise (60 mins + 20 mins GD)

**Level 1**

**Solution:** Original length

Increase in length

Longitudinal strain

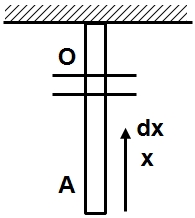
**Solution:** Cross section area of cylinder

**Solution:**

Here the increase in length is 20%. Hence

**Level 2**

**Solution:**

As tension is non-uniform along the wire, we have variable stress Consider an infinitesimal element of length at a distance from bottom end

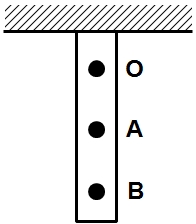
The tension at this position in the wire

Let increase in length of the small element with length.

strain

The total change in length of the wire is the sum of the changes in length of infinite such elements along the wire.

Total charge in length

1. 

**Solution:** Let be length of wire required.

Weight of wire mg

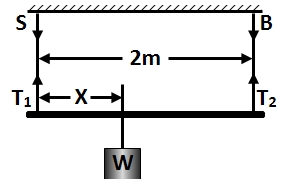
Tension and hence the stress is maximum at the topmost point O.

Here it is evident that the stress on the wire does not depend on the diameter of the wire. Hence, even if the diameter of the wire is doubled, the stress in the wire does not change. Hence the length will remain the same.

This is around 68 km length of wire. This is why the wire does not break on its own in real life situations.



**Solution:**

(a). As stresses are equal,

i.e., or …… (1)

Now for translator equilibrium of the rod,

Which in the light of eqn. (1) gives

and …….(2)

Now if x is the distance of weight W from steel wire, for rotational equilibrium of rod,

or

i.e.

(b) As strains are equal,

[as strain =]

So or i.e.,

So for translator equilibrium of rod, in the light=t of eqn. (3) yields

And for rotational equilibrium of rod

or , i.e.,

**Solution:**

stress

**Solution:**

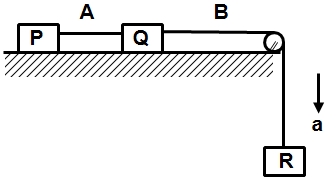
The stretching force developed in the wire due to rotation of the ball is

Let be the breaking angular velocity, than

=

**Solution:**

Let the natural length be

**Solution:**

Let the tension in the wire A and B be and respectively. Writing equations of motion of blocks P, Q and R, we get

From equations (1) And (2), we get

By equation (1) and (3),

Or

Or N

Now

We know that

**Solution:**

Given: Elongation of first wire;

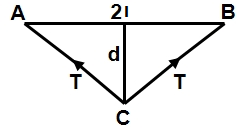
Elongation of second wire and

area of cross-section of first wire.

We know that Young’s modulus for the wire material is,

Since both the wires are similar and subjected to the same load, therefore,

**LEVEL 3**

**Solution:**

Increase in length = (AC+CB)-2

=

Longitudinal stress =T/,

(where r is radius of wire).

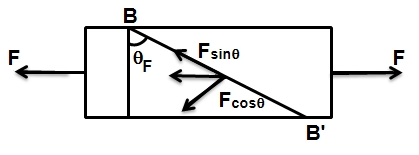
Longitudinal strain=

Now, Young’s modulus = or,

∴

If d<<, then

∴

**Solution:** The bar of cross sectional area is shown in the fig. Imagine a cut along the plane. Since this section is in equilibrium, the bar on the left must have been exerting a horizontal force uniformly to the left. This force has been resolved in two components, cos (normal to the plane) and sin (along the plane).

(a) Tensile stress=

Here Normal force=

And area of cross section=

Tensile stress =

(b) Shearing stress =

=

=

(c) From part (a), the tensile stress is given by

=

This will be maximum when,

(d) Shearing stress=

This will be maximum when

, ,

#### Homework (60 mins + 20 MINS GD)

##### LEVEL 1

**Solution:**

Here we are given that,

Length

Diameter

Hence radius

Area

Now, Stress

Strain

Young

**Solution**: Original length

Increase in length

Longitudinal strain

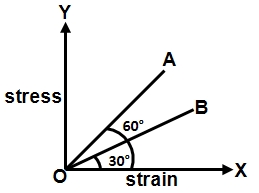
**Solution :**

Here, the force on the wire is equal to the weight of the wire hence

Stress

Stress=

##### LEVEL 2

**Solution:** Y is clearly the slope of the stress strain graph since Stress = Y strain

**Solution:**

Let the extension of wire be X for an external force F.

Stress=

This can be written as F = YAX/L

##### Equivalent force constant =



**Solution:**

1. For a straight line graph, the two quantities should be proportional

Where, weight hung.

Hence A is correct.

1. Stress F/A Y(Strain)

Stress . Hence B is correct.

stress strain

Hence C is correct

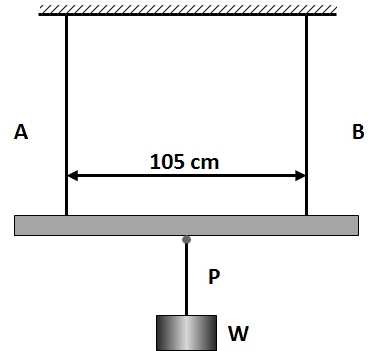
Same as A. This is also correct.

1. From 1 we have again,

Where, weight hung.

Hence D is also correct.

**Solution:**

1. Let distance of the point where and weight is suspended from wire.

Let and be the tensions in and.

Balancing torques about

As the rods have equal stress

Solving for we get :

(from A)

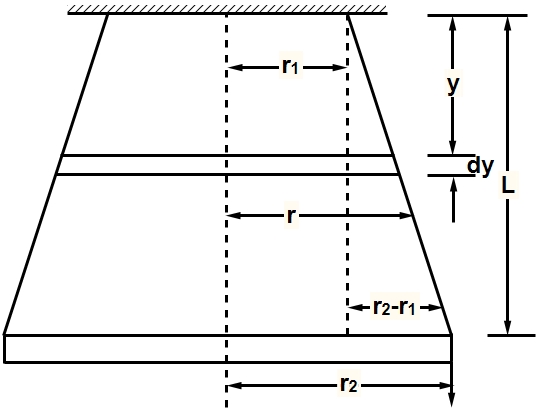
1. One equation is same i.e.

equal strain

(from A)

##### LEVEL 3

**Solution:**



The situation is shown in fig. Let be the length of the wire. Suppose and be the radii of the upper and lower ends of the wire respectively. Now the radius of the wire at a distance from the upper end is given by

Stress at a distance is given by

Corresponding strain

Consider a very small part of the wire. Extension is given by

Total extension in the length of the wire

On interchanging the ends, the change in length remains the same.

**Solution:** Sol: Stretching force=tension in the wire

=4kg wt. =4000980 dyne

Cross sectional area of wire ==

Tension per unit area=

Let the extension produced is cm

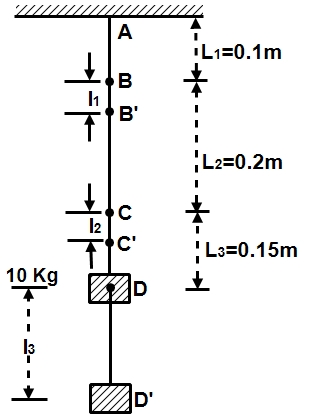
Linear strain = =

We know that

Young’s modulus =

Linear strain =

**Solution:**

The situation is shown in fig

We know that

Displacement of B

Displacement of C

Displacement of D

## Elasticity - II

#### Pre - reading EXERCISE (10 mins + 5 mins GD)

**Solution:** Energy stored per unit volume

**Solution:**

#### In-class Exercise (60 mins + 20 mins GD)

**LEVEL 1**

**Solution:**

Here atm

litre

litre

**Solution**:

Work done

**Solution:** Shear stress

Shear strain

(Shear modulus)

**Solution :** Volume stress

Volume strain

(bulk modulus)

**LEVEL 2**

**Solution:** Energy stored per unit volume = . And,

.

Therefore,

**Solution:**

This case is similar to a wire elongating longitudinally. There we have seen the tension in the wire follows a relation, just like a spring. Similarly, this behaves like a spring where

From our equation,

Hence, since torque is constant.

Therefore,

**Solution:** The increase in energy of a brass bar will be equal to work done for compressing it. The work done is given by

Where =force applied on the bar and

=increase in its length.

We know that

Here, ,

=joules

**Solution:**

We know that

Elastic energy per unit volume =

Here, strain =

And

Now , elastic energy per unit volume

**Solution:** Consider a wire of length and area of cross-section. Let be the increase in the length when a stretching force is applied. The workdone in increasing the length is given by

.

The total work done in increasing the length is given by

…………..(1)

We know that

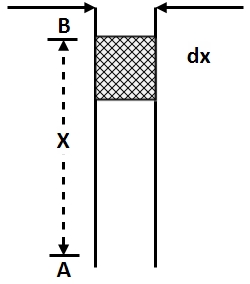
∴ …………….(2)

Substituting the value of from equation (2) in equation (1), we get

Substituting the given values, we have

= 1.066 joules.

**LEVEL 3**

1. 

**Solution:** Consider an element of wire of length at a height from

the ground as shown in figure.

Load on =weight of the part

=

If be the elongation of element then

Stretched length =

=

Entire length =

=

=

Increase in length = 0.3675m.



**Solution:** At surface of ocean atm

Or, volume has mass

Changes in pressure atm atm

Change in volume,

New density of water

Water gets compressed by about at the ocean bed due to very high pressure.

#### Homework (60 mins)

##### LEVEL 1

**Solution:**

Stress=

The shear is on the area where shear force acts, which is the curved surface area of the punched material. Hence area =

**Solution:**

Where F is the elastic force and s is the elongation.

Hence

Hence,

If we compare this expression with that of a spring , then

Hence, the work done =

**Solution:**

We know that, the extension produced in case of a cylindrical wire is directly proportional to the length (L) of the wire and inversely proportional to the area of cross-section,

Hence,

So for large extension we need longer wire but of lesser cross-section area,

Therefore, d is the correct answer.

##### LEVEL 2

**Solution:**

Young’s modulus of elasticity is given by

Substituting the values, we get

Now,

**Solution:**

We have seen that the restoring force is given as,

In equilibrium, we have seen that,

Applied torque Restoring torque.

Radians

**Solution:** Work done in producing the extension

##### LEVEL 3



**Solution**: a. Consider an element of length at a distance from the fixed end. Let be the energy stored in the element of length. Then

b.



**Solution:**

Here strain

If A be the area of cross section of the wire, then

After falling the mass through a distance 1.5 m, the velocity is given by

Now the wire will be stretched. Let be the increase in length, than the strain is given by ( /1.5)

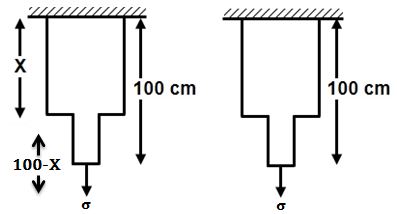
Now work done

=19.6 joule

Solving it for and taking positive value, we get =0.33 so the mass comes to rest at 1.5+0.33=1.83m below A.

**Solution:**

Maximum stress lies in stepped bar in the portion of lesser area

For the stress in lesser area, the stress in large cross-section

Strain energy of stepped bar –

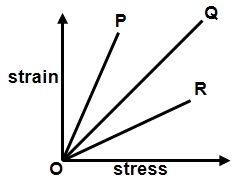
Stain energy of uniform bar

As per given condition

**Objective Questions:**

**Solution:** if radius of the wire is doubled than increment in length will become times i.e.

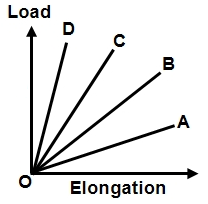
**Solution:** If density of the material increase then more force (stress) is required for same deformation i.e. the value of Young’s modulus increase.

1. 

**Solution:** As stress is shown on x-axis and strain on y-axis

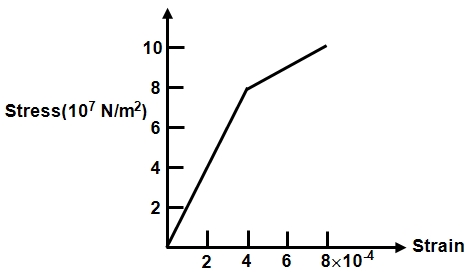
So we can say that

So elasticity of wire P is minimum and of wire R is maximum.

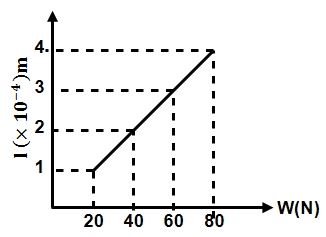
1. 

**Solution:**

i.e. for the same load , thickest wire will show minimum elongation. So graph D represent the thickest wire

1. 

**Solution:** Young’s modulus is defined only in elastic region

**Solution:** Young’s modulus is

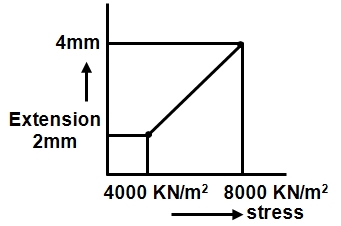
i.e. graph is a straight line passing through origin, the slope of which is .

Slope

**Solution :**

Here, since

And for displacement we have,

1. 

**Solution:** From the definition of Young’s modulus,

Slope of curve

given

**Solution:** Energy stored per unit volume =

**Solution:** Potential energy stored in the rubber cord catapult will be converted into kinetic energy of mass.

**Solution:** Bulk Modulus

**Solution:** If side of the cube is L then



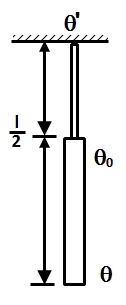
**Solution:**

Pa.

**Solution:** Twisting couple

If material and length of the wire A and B are equal and equal twisting couple are applied then

**Solution:** shearing strain





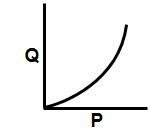
**Solution:**

⇒

**Solution:**

**Solution:** Energy per unit volume =



**Solution:** Graph between applied force and extension will be straight line because in elastic range.

Applied force extension

But the graph between extension and stored elastic energy will be parabolic in nature as,

**Solution:**

Now as, or

..... (i)

From Eq. (i)